Part VII

第七部分

Causal Sets

因果集

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The Causal Set Approach to the Problem of Quantum Gravity

量子引力问题的因果集方法

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Abstract

摘要

Causal set theory (CST) is an approach to the problem of quantum gravity based on the twin hypotheses of fundamental spacetime discreteness and primitivity of the spacetime causal relation. We situate the chapters in the CST section of the Handbook of Quantum Gravity within the landscape of research in CST.

因果集合论 (CST) 是解决量子引力问题的一种研究方案，其基于两个核心假设: 时空从根本上是离散的，且时空因果关系具有本原性。我们将《量子引力手册》CST 章节中的各章内容定位在因果集合论的研究格局中。

Keywords

关键词

Causal set theory · Causal order · Discreteness · Partial order · Passage of time - Path integral - Quantum gravity - Quantum measure theory

因果集合论 · 因果序 · 离散性 · 偏序 · 时间流逝 - 路径积分 - 量子引力 - 量子测度论

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Introduction

引言

The causal set approach to the problem of quantum gravity is based on a novel physical concept: a discrete order. This concept is postulated by the theory to be the deep structure of spacetime. Since its origins in the late 1970s and 1980s [1-3], causal set theory (CST) has grown into a program that encompasses many aspects of fundamental physics from the foundations of quantum theory to cosmology. Sorkin's early writings [4-6] remain excellent introductions to CST. For a more up-to-date review, see [7]. The chapters in the causal set section of the Handbook of Quantum Gravity represent several of the main areas of research and can serve as an introduction for nonexperts who would like to learn about this quantum gravity approach.

解决量子引力问题的因果集方法基于一个全新的物理概念: 离散序。该理论将这一概念假设为时空的深层结构。自 20 世纪 70 年代末至 80 年代诞生以来 [1-3]，因果集理论 (CST) 已经发展成为一个涵盖基础物理学诸多方面的研究纲领，研究范围从量子理论基础到宇宙学。索金的早期著作 [4-6] 至今仍是了解因果集理论的优秀入门资料。若需更前沿的综述，可参见 [7]。《量子引力手册》因果分篇的各章涵盖了多个主要研究方向，可供想要了解这一量子引力方法的非专业人士作为入门学习。

Spacetime discreteness, whether postulated ab initio or derived within the theory, is not unique to causal set theory; indeed it might be claimed that the majority of quantum gravity approaches today have or hint at some sort of discrete character. What sets causal set theory apart from other approaches is that, together with discreteness of spacetime, it insists that the spacetime causal order, of all continuum spacetime structures, is the most fundamental and survives in the deep theory as a primitive concept of order. The marriage of these twin hypotheses of discreteness and order then has an essentially unique offspring as the proposed kinematical basis of the theory: a discrete order or causal set or causet (for short). Further, this marriage seems to leave us no choice when it comes to striking an attitude to the foundations of quantum theory suitable for quantum causal set dynamics. A path integral, or sum-over-histories, approach to quantum foundations is the only one that - among currently existing approaches - seems capable of dynamically accommodating both the discreteness and the essential nonlocality [8-11] of causal sets.

无论离散性是理论从头假设还是推导得出，它都并非因果集理论独有；实际上可以说，当今大多数量子引力方法都具有或暗示了某种离散特征。因果集理论区别于其他方法的特点是，在假设时空离散性的同时，它主张在所有连续时空结构中，时空因果序是最基础的，并作为原生序概念保留在深层理论中。离散性与因果序这两个核心假设结合，自然而然得到了唯一的理论运动学基础: 离散序，即因果集 (简称 causet)。此外，这种结合也为适配量子因果集动力学的量子理论基础定下了唯一方向。在现有方法中，量子基础的路径积分 (即历史求和) 方法是唯一能够在动力学层面同时容纳因果集的离散性与本质非定域性 [8-11] 的方案。

The original paper by Bombelli et al. [3] states:

Bombelli 等人的原创论文 [3] 中有如下表述:

In the schematic evolution of Taketani and Sakata, a physical theory passes through three stages: an initial stage in which a particular ”substance”, or type of matter, presents itself in a characteristic group of phenomena; a second stage in which the new substance in question is clearly discerned in relation to the phenomena; and a final stage in which the comprehensive dynamics characterizing this substance is understood. In contrast, quantum gravity is forced to skip virtually the whole of the first stage, and tackle the second and third stages together, hoping that the resulting theory will enable us to recognize with hindsight what features of already-known physics can provide its ”phenomenology.”

根据武谷与坂田的理论发展阶段模型，一个物理理论会经历三个阶段: 第一阶段是特定“实体”(即物质类型) 在一组特征现象中显现；第二阶段是该新实体与相关现象得到明确区分；第三阶段则是厘清描述该实体的完整动力学。而量子引力实际上不得不跳过几乎整个第一阶段，同时推进第二与第三阶段，只能寄希望于最终得到的理论能让我们事后复盘，识别出已有物理中哪些特征可以为其提供“现象学基础”。

Subsequently, Sorkin [4, 5] has continued to use the three-way categorization of work in causal set theory (CST) in terms of kinematics (the properties of the substance - broadly construed - that the theory is about), dynamics (the laws of motion - very broadly construed - of the kinematical substance), and phenomenology (the observable ”surface” phenomena whereby the ”deep structure” of the substance and its dynamics reveals itself to us). These categories, though they overlap, remain very useful, and we will organize this introductory article along their lines by situating the chapters of the causal set section of the handbook within them: kinematics in section ”Kinematics,” dynamics in section ”Dynamics,” and phenomenology in section ”Phenomenology.” Today, the three categories are joined by work in mathematics and work in philosophy which intertwine with all three and which bring with them new perspectives and new expertise: section ”Mathematics and Philosophy.”

此后，索金 [4, 5] 一直沿用三分法划分因果集理论 (CST) 的研究工作: 运动学 (即理论研究对象——广义上的实体——的性质)、动力学 (即运动学实体的广义运动定律)、现象学 (即可观测的“表层”现象，实体的“深层结构”及其动力学通过这些现象向我们显现)。这些分类虽有重叠，却仍十分实用，我们这篇引言也将遵循该框架展开，将手册因果集部分的各章归入对应分类:“运动学”部分介绍运动学相关研究，“动力学”部分介绍动力学相关研究，“现象 ology”部分介绍现象学相关研究。如今，除这三类之外，数学与哲学研究也融入了所有方向，带来了新视角与新专业成果，因此增设“数学与哲学”部分。

Kinematics

运动学

In causal set theory, continuum Lorentzian spacetime as we know it in general relativity will be emergent. We can separate two different aspects of this emergence, what we will call kinematical emergence and dynamical emergence. We will discuss dynamical emergence in the next section. A crucial kinematical emergence question is whether, given a continuum spacetime (M, g) , there exists a causal set $(C, \prec)$ that can recover the geometry of (M, g) approximately, and essentially uniquely, at scales larger than the discreteness scale (assumed to be Planckian). When this is the case, we say that spacetime (M, g) is a continuum approximation to causet $(C, \prec)$ and that the causet is manifold-like. In anticipation of the quantum dynamics of causal sets as a sum over histories, we need Lorentzian geometries more generally - not just solutions of the Einstein equations - to be recoverable from causal sets in order that the quantum sum over causets in the fundamental theory does justice to the heuristic of the quantum gravitational path integral as a sum over geometries, the majority of which are ”off shell.”

在因果集合论中，我们在广义相对论中熟知的连续洛伦兹时空是演生的。我们可以将这种演生分为两个不同的方面，我们分别称之为运动学演生和动力学演生。我们将在下一节讨论动力学演生。一个关键的运动学演生问题是: 给定一个连续时空 (M, g) ， 是否存在一个因果集合 $(C, \prec)$ ， 可以在大于离散尺度 (假定为普朗克尺度) 的尺度上，近似且基本唯一地重构出 (M, g) 的几何? 当满足该条件时，我们称时空 (M, g) 是因果集 $(C, \prec)$ 的连续近似，且该因果集具有类流形性。由于因果集的量子动力学是历史求和，我们需要更一般的洛伦兹几何——而不只是爱因斯坦方程的解——能够从因果集重构出来, 这样基础理论中对因果集的量子求和才能符合量子引力路径积分作为几何求和的启发式思路，其中大多数几何都是“离壳”的。

As laid out by Bombelli et al. in their original paper [3], the question of kinematical emergence begins with a proposal for the discrete-continuum correspondence in the theory. CST proposes that a causal set, $(C, \prec)$, recovers the GR spacetime (M, g) if there exists a Planck-scale faithful embedding [3], i.e., an injective map $\phi : C \hookrightarrow M$ satisfying the following conditions:

正如 Bombelli 等人在原始论文 [3] 中提出的那样，运动学演生问题从该理论中离散-连续对应关系的一个提议开始。因果集合论 (CST) 提出，若存在普朗克尺度的忠实嵌入 [3]，即存在满足以下条件的单射映射 $\phi : C \hookrightarrow M$ ， 则因果集 $(C, \prec)$ 可以重构出广义相对论时空 (M, g) :

(i) (Planck-scale uniform): The number of causal set elements embedded in any sufficiently large, physically nice region of M is approximately equal to the spacetime volume of the region in fundamental, Planckian scale, volume units.

(i)(普朗克尺度均匀): 嵌入到 M 任意足够大、物理性质良好的区域中的因果集元素数量，以基础普朗克尺度体积单位计算，近似等于该区域的时空体积。

(ii) (Order-preserving): Elements x and y of C are ordered, $x \preceq y$, if and only $\phi(x) \in J^-(\phi(y))$.

(ii)(保序): C 的元素 x 和 y 满足有序关系 $x \preceq y$ 当且仅当 $\phi(x) \in J^-(\phi(y))$.

(iii) The characteristic distance over which the continuum geometry (M, g) varies appreciably is everywhere much greater than the Planck length/time.

(iii) 连续几何 (M, g) 发生明显变化的特征距离，处处远大于普朗克长度/普朗克时间。

The condition of ”physically nice” can be taken to mean that the region contains large, approximately flat causal intervals (causal diamonds) and that it has no Planck-scale features such as very wiggly boundaries.

“物理性质良好”的条件可以理解为: 该区域包含大的、近似平坦的因果区间 (因果钻石)，且不包含普朗克尺度的特征，比如非常起伏的边界。

A central task of causal set kinematics is to prove that the discrete-continuum correspondence of Bombelli et al. holds water: that, when there is a faithful embedding, the causal set does indeed contain enough information to recover the continuum spacetime topology and geometry at large scales essentially uniquely. This is known as the Hauptvermutung or central conjecture of causal set theory [12-14].

因果集运动学的核心任务是证明 Bombelli 等人提出的离散-连续对应关系是成立的: 即当存在忠实嵌入时，因果集确实包含足够的信息，在大尺度上基本唯一地重构出连续时空的拓扑和几何。这就是因果集理论的 Hauptvermutung，即中心猜想 [12-14]。

One type of evidence for the Hauptvermutung is the direct recovery of topological and geometrical information from a faithfully embeddable causal set (see [15-21] for example). This type of work is epitomized by the recovery of continuum spacetime dimension from a faithfully embeddable causal set - dimension was the first continuum quantity to be given a discrete form [2] - and is surveyed in the handbook - Chap. 64, ”Estimating the Manifold Dimension of Causal Sets” by F. Ashmead and D. Reid. The chapter describes the strategy that is used in much of the literature on causal set kinematics: causal sets are studied that are known to be faithfully embeddable in (M, g) by construction because each is the outcome, C , of a Poisson point process in M of Planckian intensity with an order relation $\prec$ that is the restriction of the spacetime causal order of (M, g) to C . Then, the manifold (M, g) is put aside and dimension information - or other continuum quantity - is sought from $(C, \prec)$ alone. Ashmead and Reid cover different dimension estimators in flat and curved spacetime, and numerical results probe the accuracy and efficiency of the various estimators, showing that large-scale spacetime dimension can be recovered from the order.

支持中心猜想的一类证据是，可从可忠实嵌入的因果集中直接重构拓扑和几何信息 (例如见 [15-21])。这类工作的典型代表就是从可忠实嵌入的因果集中重构出连续时空维度——维度是第一个被给出离散形式的连续量 [2]，相关研究综述收录于手册第 64 章，即 F. Ashmead 和 D. Reid 撰写的《因果集的流形维度估计》。该章描述了现有多数因果集运动学文献采用的研究策略: 研究的因果集按构造就已知可忠实嵌入 (M, g) ，因为每个因果集都是 M 中普朗克强度泊松点过程的结果 C ，其序关系 $\prec$ 就是 (M, g) 的时空因果序对 C 的限制。随后研究者将流形 (M, g) 放在一边，仅从 $(C, \prec)$ 出发寻找维度信息或其他连续量。Ashmead 和 Reid 介绍了平坦时空和弯曲时空中不同的维度估计量，数值结果探测了各估计量的精度和效率，表明大尺度时空维度可以从序关系中重构出来。

Dynamics

动力学

The framework for the quantum dynamics of causal sets is the sum-over-histories or path integral, and there are two main regimes of interest. In one regime, the deep quantum regime, there is no continuum approximation, and we will want to use the theory to answer questions that cannot be answered within continuum GR such as ”What was going on just before the big bang?” and ”What happens at the singularity inside a black hole horizon?”

因果集量子动力学的框架是路径积分即历史求和，目前主要有两类感兴趣的研究区域。其中一类是深量子区域，该区域不存在连续近似，我们需要用该理论回答连续广义相对论无法解答的问题，例如“大爆炸发生之前究竟是什么状况?”以及“黑洞视界内部奇点处会发生什么?”

The second regime of interest is the continuum regime in which GR is recovered by the theory and the physics is well-described approximately by a continuum GR spacetime with matter propagating on it. The question of dynamical emergence of GR in causal set theory then is whether, in this second, continuum regime of the dynamics, causal sets faithfully embeddable in GR solutions arise and are predictions of the theory. The exciting potential of causal set dynamics is that in this regime it can hope eventually to explain certain properties of continuum spacetime, such as its dimension and topology.

另一类感兴趣的研究区域是连续区域，该区域中广义相对论可由该理论还原，物理过程可以很好地近似描述为物质在连续广义相对论时空中传播。那么因果集理论中广义相对论的动力学涌现问题就是: 在动力学的这第二个连续区域中，可忠实嵌入广义相对论解的因果集是否会产生，且该理论能否给出对应预言。因果集动力学令人振奋的潜力在于，在该区域它最终有望解释连续时空的维度、拓扑等特定性质。

Work on the dynamics of causal sets today falls into two rough categories which we call state sum models and growth models. The handbook - Chap. 67, ”Computer Simulations of Causal Sets” by L. Glaser describes state sum models for causal set quantum gravity. The partition function for CST is a sum over all causal sets of fixed cardinality N , each weighted by the action term $\exp(iS(C)/\hbar)$ where $S(C)$ is a suitable discrete action for causal set C . One choice for $S(C)$ is the Benincasa-Dowker-Glaser (BDG) action [18, 19, 22]. While there has been some progress in understanding the contribution to the partition function from the entropically dominant class of causal sets (see Chap. 69, ”Toward the Emergence of Continuum Spacetime in Causal Set Theory”), it is not straightforward to do computations with the complex weights in the partition function. Instead, one can work with an analytical continuation of the path integral which renders it into a statistical theory. This partition function and associated expectation values of order invariants can then be studied using Monte Carlo Markov Chain simulations. In order to simplify the computations further, these models are often studied with a dimensional restriction both in the action as well as in the sum. In this chapter, the 2D state sum model is described in detail. Without matter it exhibits a first-order phase transition from a continuum to a non-continuum phase. This transition is tracked using order invariants like the Myrheim-Myer dimension (see ▶ Chap. 64, ”Estimating the Manifold Dimension of Causal Sets”) and the BDG action itself. The Hartle-Hawking wave function which is calculated semi-analytically exhibits sharp peaks in the two phases. The phase structure is shown to become richer with the introduction of Ising spins, but the transitions continue to be first order.

当前因果集动力学的研究大致可分为两类，我们称之为状态和模型与增长模型。本手册第 67 章由 L. Glaser 撰写的《因果集的计算机模拟》介绍了因果集量子引力的状态和模型。因果集理论 (CST) 的配分函数是对所有固定基数 N 的因果集求和，每个因果集由作用量项 $\exp(iS(C)/\hbar)$ 加权，其中 $S(C)$ 是因果集 C 的合适离散作用量。$S(C)$ 的一个选择是贝宁卡萨-道克-格拉泽 (BDG) 作用量 [18, 19, 22]。尽管目前在理解熵主导类别因果对对配分函数的贡献方面已经取得了一些进展 (参见第 69 章《因果集理论中连续时空的涌现之路》)，但使用配分函数中的复权重进行计算并不简单。我们可以转而对路径积分进行解析延拓，将其转化为统计理论，之后就可以利用马尔可夫链蒙特卡洛模拟研究该配分函数以及序不变量的相关期望。为了进一步简化计算，研究这些模型时通常会在作用量和求和过程中都加上维度限制。本章详细介绍了二维状态和模型。不存在物质时，该模型展现出从连续相到非连续相的一级相变。我们使用梅尔海姆-迈尔维度 (参见 ▶ 第 64 章《因果集流形维度估计》) 和 BDG 作用量本身来追踪该相变。半解析计算得到的哈特-霍金波函数在两个相中都存在尖锐峰。引入伊辛自旋后，相结构会变得更加丰富，但相变仍然保持为一级相变。

The handbook - Chap. 69, ”Toward the Emergence of Continuum Spacetime in Causal Set Theory” by A. Mathur sets out the challenge posed by the fact that the number of causal sets of cardinality N grows super-exponentially with N . The entropic weight of this huge class of non-manifold-like causets in the path sum therefore threatens the existence of a semiclassical regime. The results of Kleitman and Rothschild (KR) [23], followed by Dhar [24], tell us that in the limit as the cardinality N tends to infinity, the entropically dominant orders are nothing like spacetime and threaten to overwhelm any phenomenologically interesting contributions to the causal set path sum. Mathur’s chapter describes work, starting with that by Loomis and Carlip [25], showing how the contributions of the KR causets to the path sum are suppressed to leading order [26,27]. This result relies on the BDG action being nonlocal and gives hope that there can be a continuum regime in causal set quantum gravity.

本手册第 69 章由 A. Mathur 撰写的《因果集理论中连续时空的涌现之路》阐述了一个核心挑战: 基数为 N 的因果集数量随 N 呈超指数增长。路径求和中这类海量非流形因果集的熵权重会对半经典区域的存在构成威胁。Kleitman 和 Rothschild(KR)[23] 以及后续 Dhar[24] 的研究结果表明，在基数 N 趋于无穷的极限下，熵主导的序完全不同于时空，会淹没因果集路径求和中任何唯象上有意义的贡献。Mathur 的章节介绍了从 Loomis 和 Carlip[25] 开始的相关工作，研究表明 KR 因果集对路径和的贡献在领头阶会被压制 [26,27]。该结果依赖于 BDG 作用量的非局域性，也给因果集量子引力存在连续区域带来了希望。

The other paradigm for causal set dynamics is that of growth models. The original paper on growth models by Rideout and Sorkin [28] introduced to physics the new concept of the process of the birth of spacetime atoms and of a causal set universe that grows via this birth process, element by element in a stochastic process called classical sequential growth (CSG). These stochastic models are the basis of the proposal that the birth of spacetime atoms is the physical passage of time in CST [29] which has led to a proposal for a solution of the hard problem of consciousness within CST [30]. The measure theoretic formulation of these models provides a template for covariant observables as covariant events [31].

因果集动力学的另一类范式是增长模型。Rideout 和 Sorkin 在关于增长模型的原始论文 [28] 中向物理学界引入了一个新概念: 时空原子不断诞生，因果集宇宙通过这一诞生过程逐元增长，该随机过程被称为经典序增长 (CSG)。这些随机模型是“时空原子的诞生就是因果集理论中时间的物理流逝”这一猜想的基础 [29]，该猜想还推动了在因果集理论框架内解决意识难题的方案 [30]。这些模型的测度理论表述为协变可观测量 (即协变事件) 提供了研究模板 [31]。

CSG models are defined by a sequential process that depends on a linear labeling of the causet elements: a gauge. The handbook - Chap. 71, ”Covariant Growth Dynamics” by S. Zalel is a survey of the project within causal set theory to define growth dynamics for causal sets that make no reference to labels and that are defined in terms of only gauge-independent concepts from the very beginning. Such models would be explicitly covariant growth models, and the project is very ambitious given that, as far as we know, there is no other case in physics in which the dynamics of a gauge theory is successfully defined directly in terms of the gauge-independent variables. The discreteness of causal sets, however, seems to open the door to this possibility. The central concept is that of covtree which is a tree, each of whose nodes is a covariant property of the growing causal set. The novelty that the chapter describes is that dynamical models could be random walks directly defined as happening on covtree. The dynamics then gives an account of a growing amount of physical information about the discrete universe. The framework for covariant growth therefore exists; the challenge is to define physically motivated models, where a model is a choice of transition probabilities for the edges of the tree.

CSG 模型由一个依赖于因果集元素线性标记 (规范) 的顺序过程定义。本手册第 71 章由 S. Zalel 撰写的《协变生长动力学》概述了因果集理论内的一个研究项目: 该项目旨在为因果集定义不依赖于标记、从一开始就仅用规范无关概念构建的生长动力学。这类模型即为显式协变生长模型，该项目极具挑战性，因为据我们所知，物理学中尚无其他案例成功直接用规范无关变量定义规范理论的动力学。但因果集的离散性似乎为这种可能性打开了大门。该理论的核心概念是协变树 (covtree): 这是一棵树，其每个节点都对应生长中因果集的一个协变性质。本章所述的创新点在于，动力学模型可以是直接定义在协变树上的随机游走。该动力学由此可以描述离散宇宙中不断增长的物理信息量。因此协变生长的框架已经建立；当前的挑战是定义具备物理动机的模型，这类模型需要为树的边选定跃迁概率。

Growth models have a conceptual advantage over the state sum models. The different phases of a state sum model - e.g., ”geometric” phases in which Lorentzian spacetime is a good approximation and non-geometric phases - fit most naturally into the paradigm of statistical mechanics in which the temperature of an environment and/or the volume of the system can be considered to be parameters or conditions that can be changed from outside the system, which change drives phase transitions. In a cosmological context in which there’s nothing external to the system, however, the transition between such phases would have to be driven, ultimately, by something internal to the universe. In standard cosmology that ”something” is time: the universe is expanding and so as time passes, the temperature falls, and matter can undergo phase transitions driven by this internal change.

生长模型相对态叠加模型具备概念优势。态叠加模型的不同相——例如洛伦兹时空能给出良好近似的“几何相”与非几何相——最自然地契合统计力学范式: 在该范式中，环境温度和/或系统体积可被视为可从系统外部改变的参数或条件，这类改变驱动相变。但在宇宙学语境下，系统不存在外部，这类相变最终必须由宇宙内部的某种因素驱动。在标准宇宙学中，这种“因素”就是时间: 宇宙不断膨胀，随着时间推移温度下降，物质可以发生由这种内部变化驱动的相变。

In quantum gravity, however, continuum ”time” itself is supposed to be emergent in a geometric or semi-classical phase, and so it can’t be continuum time that drives the change from one phase of quantum gravity to another. In causal set theory, in the deep theory, there is indeed no continuum time, but there is still a concept of physical order. And in a growth model, the order of the growing causal set is the physical order of the birth of the causal set elements in the growth. So, there is physical growth even when there is no continuum time, and phase transitions and renormalization of physical constants can be dynamical [32,33].

但在量子引力中，连续“时间”本身本就应当是几何相或半经典相中的涌现概念，因此不可能由连续时间驱动量子引力从一个相转变到另一个相。在因果集理论的基础理论中，确实不存在连续时间，但仍然存在物理序的概念。而在生长模型中，生长中因果集的序就是因果集元素在生长过程中诞生的物理序。因此，即便不存在连续时间，依然存在物理生长，相变和物理常数的重整化都可以是动力学过程 [32,33]。

To summarize, the existing state sum models are quantum, though they require restriction of the full history space of all causal sets to be tractable. The best studied growth models are classically stochastic, the CSG models. Although there is a class of quantum growth models - complex transitive percolation - where the probability parameter p becomes complex [34], their physical status is not clear. The development of a physically motivated quantum sequential growth model is an important frontier in causal set theory. The two paradigms of state sum and growth are not in conflict: it is possible that the insights of the state sum models will eventually find their home within a quantum growth model, as different phases in the models are realized as different epochs in the growing causal set.

综上，现有的态叠加模型均为量子模型，只是它们需要限制全体因果集的完整历史空间才能保持可处理性。研究最多的生长模型是经典随机的 CSG 模型。尽管存在一类量子生长模型——复杂渗流模型，其中概率参数 p 变为复数 [34]，但其物理意义尚不明确。构建具备物理动机的量子顺序生长模型是因果集理论的一个重要前沿方向。态叠加与生长两种范式并不矛盾: 态叠加模型的洞见最终完全可以融入量子生长模型，因为模型中的不同相对应于生长中因果集的不同纪元。

As mentioned in the Introduction, one aspect of the causal set quantum gravity program in the dynamical arena is to develop the path integral or histories approach to quantum foundations. There is no chapter in the causal set section on quantum foundations, and here we briefly sketch some of the work in this area motivated by CST. The work includes the challenge of providing the path integral with a physical interpretation and solving the measurement problem in the histories approach. In the histories approach to quantum foundations due to Sorkin, quantum theory is considered to be a species of measure theory in which the measure does not satisfy the Kolmogorov sum rule but a generalization of it: the quantum sum rule [35].

正如引言所述，动力学领域的因果集量子引力计划的一个方向是发展量子基础的路径积分或历史方法。本手册因果集部分没有专门章节讨论量子基础，在此我们简要概述受因果集理论 (CST) 启发的该领域相关工作。相关工作包括为路径积分提供物理解释，以及解决历史方法中的测量问题。在 Sorkin 提出的量子基础历史方法中，量子理论被视为一种测度论，其测度不满足柯尔莫哥洛夫加和规则，而是满足它的推广形式: 量子加和规则 [35]。

The quantum measure is given by a path integral or more accurately a double path integral. In quantum measure theory (QMT), the quantum system is closed, there are no external observers, and the interpretation is based fundamentally on the central concept in a measure theory: event. An event is, in technical terms, a measurable subset of the space of all the histories over which the sum over histories is done. Conceptually, an event can be thought of as a property of a history (the common and defining property of the histories in the event) or equivalently as something that might happen in the theory. D. Reid has suggested the term occurable as a synonym for event to emphasize its contingent nature. Every event either occurs or doesn't occur, and the physical world in a QMT is the complete list of all the events that do occur. What sets QMT apart from classical stochastic processes is that, while the events themselves form a Boolean algebra, the pattern of occurrences and non-occurrences of the events does not necessarily respect that Boolean structure [36- 38], and the physical rules of inference in a QMT will not conform to Boolean logic. To illustrate this, consider an event, A , and its complement/negation $\bar{A}$. Both A and $\bar{A}$ are events in the Boolean event algebra, and their union is the whole set of histories. Classical rules of inference imply that if A occurs, then $\bar{A}$ does not occur and vice versa: exactly one of A and $\bar{A}$ occurs. In QMT, however, it is possible that both or neither of A and $\bar{A}$ occurs [39-41].

量子测度由路径积分给出，更准确地说是双路径积分。在量子测度论 (QMT) 中，量子系统是闭合的，不存在外部观测者，其诠释从根本上基于测度论的核心概念: 事件。从技术角度来说，事件是对所有历史做历史求和的历史空间中的可测子集。从概念上，事件可被理解为某条历史的属性 (即事件内所有历史共有的定义属性)，或者等价地说，是理论中可能发生的某事。D. 里德提出用术语“可发生事件”作为事件的同义词，以强调它的或然性质。每个事件要么发生要么不发生，量子测度论中的物理世界就是所有已发生事件的完整列表。量子测度论区别于经典随机过程的点在于: 尽管事件本身构成布尔代数，但事件发生与不发生的模式不一定符合布尔结构 [36- 38]，因此量子测度论中的物理推理规则也不遵循布尔逻辑。举例来说，考虑事件 A ，以及它的补/否定事件 $\bar{A}$ 。A 和 $\bar{A}$ 都是布尔事件代数中的事件，二者的并集是整个历史集合。经典推理规则认为，若 A 发生则 $\bar{A}$ 不发生，反之亦然: A 和 $\bar{A}$ 中恰好有一个发生。但在量子测度论中，A 和 $\bar{A}$ 有可能都发生，也有可能都不发生 [39-41]。

The interpretation of QMT is a work in progress, but it is a one-world interpretation: every event either happens or doesn't happen, definitely, in the one world. A solution of the measurement problem would be to discover a successful interpretational scheme in which the pattern of occurrences and non-occurrences of macroscopic events - such as pointer positions - is always Boolean aka classical. This would prove that exactly one pointer position happens and the others do not happen. This has already been achieved within one particular QMT scheme on the assumption of permanent records (Section 7 of [38]), but it remains to be seen whether or not that scheme is physically successful.

量子测度论的诠释仍在发展中，但它属于单世界诠释: 在同一个世界中，每个事件确定要么发生要么不发生。解决测量问题的思路是找到一套可行的诠释框架，使得宏观事件 (例如指针位置) 发生与不发生的模式始终符合布尔结构，也就是经典结构。这就能证明恰好有一个指针位置发生，其余都不发生。这一点已经在某套特定的量子测度论框架中，基于永久记录假设得到了证明 ([38] 第 7 节)，但该框架在物理上是否成立仍有待验证。

Technical work in QMT includes a proof that in quantum mechanics the canonical Hilbert space is physically isomorphic to the ”event Hilbert space” constructed from the path integral and the free vector space over the event algebra [42,43]. There is an ongoing effort to put the quantum measure on a firmer mathematical footing when the set of histories is infinite [34, 44, 45] .

量子测度论的技术工作包括证明: 量子力学中，标准希尔伯特空间与由路径积分和事件代数上的自由向量空间构造出的“事件希尔伯特空间”物理同构 [42,43]。目前仍有研究在尝试，当历史集合为无限集 [34, 44, 45] 时，为量子测度建立更坚实的数学基础。

Phenomenology

现象学

Quantum gravity phenomenology is a broad category. It encompasses the recovery of known physics - in this case GR and QFT - as approximations from the theory, thereby providing a deeper understanding of that known physics. However, no one would be satisfied with just that, and one also wants to make new predictions of novel and unexpected phenomena whose observation will be evidence that one is working along the right lines.

量子引力现象学是一个宽泛的范畴。它涵盖了从理论中近似得到已知物理——此处指广义相对论 (GR) 和量子场论 (QFT)——从而加深对这些已知物理的理解。但没人会就此满足，研究者还希望对新奇、意外的现象做出新预言，观测到这些现象就能证明研究方向是正确的。

The Chap. 68, ”Causal Set Cosmology” by M. Ahmed and H. Shafi reviews causal set cosmology as the foremost testing ground for the theory. Potential cosmological insights from the classical sequential growth models mentioned above are discussed. In a CSG model belonging to a certain class, with probability one, there are an infinite number of ”posts,” which would correspond in the continuum to cosmological bounces. In the era after a post, the dynamics is governed by effective renormalized coupling constants, which renormalization ”flow” has a one parameter family of fixed points, which is transitive percolation [33]. It is an open problem to identify the basin of attraction of this flow. The causal set after a post is a universe with an origin, and one can study its effective dynamics, assuming that dynamics is transitive percolation and studies what features it typically has.

第 68 章由 M. Ahmed 和 H. Shafi 撰写的《因果集合宇宙学》，将因果集合宇宙学作为该理论最重要的试验场进行综述。本章讨论了前文提到的经典顺序增长模型带来的潜在宇宙学启发。在某一类 CSG 模型中，概率为 1 地存在无穷多个“柱”，对应连续谱中的宇宙反弹。柱之后的演化阶段，动力学由有效重整化耦合常数主导，该重整化“流”存在单参数族不动点，即可递推移渗过程 [33]。确定该流的吸引域仍是一个未解决问题。柱之后的因果集合是一个有起源的宇宙，假设动力学为可递推移渗，就可以研究它的有效动力学，并分析它通常具备哪些特征。

A very important moment in the history of causal set theory was the direct detection in 1998 of the accelerated expansion of the universe [46,47]. Non-zero Λ had been previously predicted by Sorkin [5,48] as a nonlocal, quantum effect, using a heuristic argument based on basic ingredients from causal set theory [49]. Chapter 68, ”Causal Set Cosmology” describe a concrete stochastic homogeneous model [50, 51] based on Sorkin’s argument in which Λ is ”everpresent” in the sense that, though it fluctuates between positive and negative values, the absolute value of the dark energy density is typically of the order of the total energy density of the universe. Recently, different Everpresent Λ models have been tested against cosmological data sets. In [52,53] Das, Nasiri, and Yazdi investigate the model for Everpresent Λ of Ahmed, Dodelson, Greene, and Sorkin - known as Model 1 - and show that in tests involving Supernovae (SN) 1a data, a small fraction of random seeds (∼ 0.14%) for Model 1 can produce expansion histories with a similar χ^2 value to the Λ - CDM model and a smaller fraction (∼ 0.015%) can do better than Λ - CDM. However, there are discrepant results from confronting Everpresent Λ with CMB data. In [54], using a different model of Everpresent Λ - known as Model 2 - some random seeds produced expansion histories that do as well as Λ -CDM at matching CMB data, whereas in [53] no seeds did as well as Λ -CDM at matching CMB data. It remains to be worked out what differences between the models and their implementation are responsible for the different results. One challenge that all tests of Everpresent Lambda face is that there is no agreed-upon test of success or failure for a genuinely stochastic cosmological model. It is known that all current Everpresent Λ models have certain weaknesses: they are classically stochastic and not quantum, they are forced to be homogeneous, and runs of Model 1 have to be terminated if a fluctuation of Λ causes the RHS of the Friedmann equation to go negative. Work needs to be done to extend Everpresent Λ models to be quantum and inhomogeneous and to be meaningful when there are large negative fluctuations in Λ .

因果集理论发展史上一个非常重要的节点是 1998 年直接探测到宇宙加速膨胀 [46,47]。索金此前基于因果集理论的基本要素通过启发式论证，就已经预言非零的 Λ 是一种非局域量子效应 [5,48,49]。第 68 章“因果集宇宙学”描述了一个基于索金论证的具体随机均匀模型 [50, 51]，在该模型中 Λ 是“恒现的”：即尽管它在正负值之间涨落，但暗能量密度的绝对值通常处于宇宙总能量密度的量级。近年来，不同的恒现 Λ 模型已经得到宇宙学数据集的检验。达斯、纳西尔和亚兹迪在文献 [52,53] 中研究了艾哈迈德、多德尔森、格林和索金提出的恒现 Λ 模型 (即模型 1)，他们指出，在涉及 Ia 型超新星 (SN) 数据的检验中，模型 1 的一小部分随机种子 ($\sim 0.14\%$) 可以得到与 Λ-CDM 模型 χ^2 值相近的膨胀历史，更少比例 ($\sim 0.015\%$) 的种子得到的结果甚至优于 Λ-CDM 模型。然而，将恒现 Λ 与 CMB 数据对比得到了不一致的结果。在文献 [54] 中，研究者使用另一种恒现 Λ 模型 (即模型 2)，发现部分随机种子得到的膨胀历史在匹配 CMB 数据上的表现和 Λ-CDM 一样好；而文献 [53] 中没有任何种子在匹配 CMB 数据上能达到 Λ-CDM 的表现。模型本身及其实现方式的哪些差异导致了不同结果，这一问题仍有待解决。所有恒现 Λ 模型检验都面临一个挑战：对于一个本质上是随机的宇宙学模型，目前尚无一致认可的成败检验标准。已知当前所有恒现 Λ 模型都存在一定缺陷：它们是经典随机而非量子模型，被约束为均匀，且当 Λ 涨落导致弗里德曼方程右侧为负时，模型 1 的运行就必须终止。未来仍需开展工作，将恒现 Λ 模型扩展为量子非均匀模型，并解决 Λ 出现大幅负涨落时模型仍保持意义的问题。

One important development in recent years has been the construction of a scalar QFT on a causal set, as described in handbook - Chap. 70, ”Quantum Field Theory on Causal Sets” by Nomaan X. The chapter describes the Sorkin-Johnston (SJ) construction of a distinguished ground state for a free scalar field on a finite causal set, starting from the retarded Green function. The retarded Green function for a (massless or massive) scalar field on a causal set is known for a special class of manifold-like causal sets: those approximated by Minkowski and de Sitter spacetimes in two and four dimensions, as well as Riemann normal neighborhoods of certain two- and four-dimensional spacetimes. Numerical studies of the SJ states in these cases are described in detail and compared to analytic results when these are available. At small-distance scales, these show a consistent mismatch with the continuum which could be useful phenomenological signatures of new physics due to the discreteness.

近年来一项重要进展是在因果集上构建了标量量子场论，收录于手册第 70 章，由 Nomaan X 撰写《因果集上的量子场论》。该章介绍了有限因果集上自由标量场 distinguished 基态的 Sorkin-Johnston(SJ) 构造，该构造从推迟格林函数出发。对于一类特殊的类流形因果集——即由二维和四维闵氏时空、德西特时空以及某些二维和四维时空的黎曼法向邻域近似得到的因果集，其上 (无质量或有质量) 标量场的推迟格林函数是已知的。文中详细介绍了这些情况下 SJ 态的数值研究，并在有解析结果时将数值结果与解析结果进行了对比。在小尺度下，数值结果始终与连续时空结果存在偏差，这种偏差可作为离散性带来的新物理可观测唯象特征。

Another important potential phenomenological consequence of discreteness, not included in one of the handbook chapters, is the ”swerving” of particles. The random discreteness of manifold-like causal sets gives rise to a diffusion in momentum space for particle propagation, leading to potentially observable acceleration over cosmological times [55]. The search for very high energy neutrinos that are clearly cosmological in origin could be used to constrain the swerve model applied to neutrinos.

离散性的另一项重要潜在唯象效应并未收录在手册的任一章节中, 那就是粒子的“偏折”(swerving)。类流形因果集的随机离散性会导致粒子传播过程中在动量空间发生扩散，在宇宙学时间尺度上可能产生可观测的加速度 [55]。搜寻起源明确为宇宙学的超高能中微子，可用于约束适用于中微子的偏折模型。

The handbook - Chap. 66, ”Interacting Quantum Scalar Field Theory on a Causal Set” by Ian Jubb develops the extension to interacting fields of the Sorkin-Johnston theory on a finite causal set. The theory was originally defined by Sorkin as a Schwinger-Keldysh path integral in the in-in formalism for $\lambda\phi^4$ theory, and this paper describes how this is equivalent to an operator formalism in which there is a well-defined interaction picture in Tomonaga-Schwinger form. The perturbative series expansion in λ is finite, and the diagrammatic calculus for the interacting time ordered two-point function is developed.

手册第 66 章《因果集上的相互作用量子标量场论》由 Ian Jubb 撰写，将有限因果集上的 Sorkin-Johnston 理论拓展到了相互作用场。该理论最初由 Sorkin 定义, 是 $\lambda\phi^4$ 理论进动系 (in-in formalism) 下的施温格-凯尔迪什路径积分，本章介绍了该理论等价于拥有 Tomonaga-Schwinger 形式明确定义相互作用绘景的算符形式。λ 的微扰级数展开是有限的，文中还建立了相互作用时序两点函数的图计算方法。

In Chap. 72, ”Entanglement Entropy and Causal Set Theory”, Y. Yazdi reviews entanglement entropy of a scalar QFT in causal set theory. The entanglement is calculated by using an expression for the entropy of a Gaussian state of a scalar field on a Cauchy surface of a globally hyperbolic spacetime that uses only the spacetime two-point correlation function W . Throughout the chapter, the primacy of spacetime concepts and spacetime regions is explained and emphasized. When applied to the massless field on a causal set which has the 1+1 diamond as an approximation, the result for the entanglement entropy of the field in a small diamond as a subset of a larger diamond scales like the area of the small diamond and not logarithmically (which is the analogue in 1+1 of the area law). The chapter describes the resolution of this issue, explaining how extra ”cross boundary modes” are being counted by the entropy that are not present in the continuum. The chapter explains how to identify these ”non-continuum” modes and how to eliminate them from the calculation, resulting in an entropy that does match the expected result from the continuum. The chapter then describes how the technology can be applied to more general situations including entanglement entropy for disjoint causal diamonds.

第 72 章《纠缠熵与因果集理论》中，Y. Yazdi 综述了因果集理论中标量量子场论的纠缠熵。该纠缠熵通过全局双曲时空柯西面上标量场高斯态熵的表达式计算，该表达式仅用到时空两点关联函数 W 。整章阐释并强调了时空概念与时空区域的首要地位。将该方法应用于由 1+1 维菱形近似的因果集上的无质量场时，大菱形内部小菱形中场的纠缠熵结果按小菱形面积标度，而非对数标度 (对数标度才是 1+1 维面积律的对应形式)。本章介绍了该问题的解决方案，解释了熵额外计数了连续时空不存在的“跨边界模式”，说明了如何识别这些“非连续”模式并将其从计算中剔除，最终得到的熵与连续时空的预期结果一致。本章随后介绍了该方法如何应用于更一般的情况，包括不相交因果钻石的纠缠熵计算。

Chapter 65, ”On Horizon Molecules and Entropy in Causal Sets” by D. Dou reviews the progress to date in accounting for black hole entropy as a counting of so-called horizon molecules in causal set theory, by analogy with the statistical mechanics of a gas in the high temperature limit in which the entropy is dominated by the number of molecules times a temperature-dependent constant. The history of the subject is set out starting with the seminal work of Dou and Sorkin that created the concepts and ideas on which the subsequent work is based. The description of subsequent work focuses on the concepts but also contains enough technical detail to be able to understand the form that analytic calculations take in this field. The advantages and drawbacks of different proposals for horizon molecules are set out and are compared to one another.

第 65 章《因果集中的视界分子与熵》由 D. Dou 撰写，综述了因果集理论中将黑洞熵解释为所谓视界分子计数的研究进展，该解释类比了高温极限下气体的统计力学——在该极限下，熵由分子数乘以温度相关常数主导。本章梳理了该领域的研究历史，从 Dou 与 Sorkin 奠定后续研究概念与思想基础的开创性工作开始讲起。对后续工作的介绍以概念阐述为主，但也包含了足够的技术细节，以便理解该领域解析计算的形式。本章列出了不同视界分子方案的优缺点，并对各方案进行了对比。

Mathematics and Philosophy

数学与哲学

The mathematics of causal sets is a rich field as reviewed by G. Brightwell and M. Luczak [56]. As there is no handbook chapter focused on mathematics, we here pick out one example that illustrates both how causal set math is part of graph theory - a causal set is a transitive directed acyclic graph - and how the ordered character of causets makes their mathematics very different from unordered graphs. The simplest of the classical sequential growth models for causal sets is known to mathematicians as the model of random graph orders (see Section 3 of [56]) and in the physics literature as the transitive percolation models mentioned above [28]. Transitive percolation is the Lorentzian analogue of the Erdös-Rényi random graph $G(n, p)$ and has a very different behavior. $G(n, p)$ is the random graph on n vertices where each pair of vertices is an edge with probability p . For any non-zero $p < 1$, the Erdös-Rényi random graph on countably infinite vertices, $G(\infty, p)$, is almost surely isomorphic to one particular infinite graph, the Rado graph or Erdös-Rényi graph. In physics language, we would say that the corresponding stochastic process of growing an unlabeled graph - by the birth of one vertex at a time - is ”deterministic” when run to infinity, even though the graph at any finite stage of the growth will be different on different runs. Transitive percolation is related to the random graph in the following way. Take the ground set of the random graph $G(n, p)$ to be the natural numbers $\{0, 1, 2 \ldots n-1\}$. If $i < j$ and there is an edge (i, j) in G (n, p) , then put $i \prec j$ and take the transitive closure to form the random order $T(n, p)$. The two models seem close. But in contrast to the random graph, in transitive percolation with a given p , the growth dynamics is not deterministic: the process gives a probability distribution on countable past finite orders, and moreover, the distribution depends on p .

因果集数学是一个丰富的领域，G. Brightwell 与 M. Luczak 已对其做综述 [56]。由于目前没有专门聚焦该领域数学的手册章节，我们在此选取一个例子，同时说明两点: 因果集数学如何属于图论的一部分——因果集是一个传递有向无环图——以及因果集的有序特性使得其数学性质与无序图截然不同。因果集最简单的经典顺序增长模型，在数学界被称为随机图序模型 (见 [56] 第 3 节)，在物理学文献中则对应上文提到的传递渗流模型 [28]。传递渗流是埃尔德什-伦伊随机图 $G(n, p)$ 的洛伦兹类比，其行为截然不同。$G(n, p)$ 是拥有 n 个顶点的随机图，任意一对顶点之间以概率 p 连边。对于任意非零 $p < 1$ ，可数无穷顶点上的埃尔德什-伦伊随机图 $G(\infty, p)$ 几乎必然同构于某个特定无穷图，即拉多图，也叫埃尔德什-伦伊图。用物理学语言来说，对应的逐点生长无标号图的随机过程，即便有限生长阶段的图每次运行结果都不同，运行到无穷时它是“确定性”的。传递渗流与随机图的关联如下: 将随机图 $G(n, p)$ 的基集取为自然数集 $\{0, 1, 2 \ldots n-1\}$ 。如果满足 $i < j$ 且 G(n, p) 中存在边 (i, j) ，则令 $i \prec j$ 成立，再取传递闭包即可得到随机序 $T(n, p)$ 。两种模型看似相近，但与随机图不同，给定 p 的传递渗流的生长动力学不是确定性的: 该过程会在可数过去有限序上给出概率分布，且该分布依赖于 p 。

Philosophical considerations have been influential in causal set research from its earliest stages; see, for example, the influence of the Taketani scheme [57] with its three-way kinematics/dynamics/phenomenology categorization mentioned in the Introduction section ”Introduction.” There is now also a body of work by philosophers of physics in which philosophical issues of importance and interest are investigated in the rather concrete context of causal sets. In the handbook Chap. 63, ”The Philosophy of Causal Set Theory”, C. Wüthrich surveys some of these issues. One such is the question of emergence and in particular the emergence of continuum spacetime structure from a fundamentally discrete theory. The chapter argues that adherence to a position of ”spacetime functionalism” can evade a claimed contradiction that threatens to block the recovery of continuum spacetime from discrete underpinnings. As the preceding sections of this introductory article show, the question of emergence of Lorentzian spacetimes from CST, both at the kinematical and dynamical level, is one that occupies the community of causal set theorists. This is an example of an important issue of common concern with potential for helpful engagement on both sides and for collaboration [58]. Another focus of the chapter is the philosophy of time. Here, again there is interest from both causal set theorists and philosophers. One debate centers around whether and in what sense the birth process in classical sequential growth models constitutes physical becoming, or in other words a physical passage of time, as argued by causal set theorists. In the chapter Wüthrich argues, from a skeptical position, that the becoming in CSG models is analogous to ”worldline becoming” in the context of continuum GR. The debate hinges on the meaning and understanding of the concept of ”process” in physics and mathematics and is a prime example of fruitful engagement and challenge that can only be positive for the future development of the theory.

从因果集研究的最初阶段开始，哲学思考就对该领域产生了重要影响；例如引言部分“引言”提到的、分为运动学/动力学/现象学三分法的竹谷方案就对研究产生了影响 [57]。目前也有物理哲学家开展了大量研究，在因果集这一相当具体的背景下探究重要且值得关注的哲学问题。在手册第 63 章“因果集理论的哲学”中，C. Wüthrich 综述了部分这类问题。其中一个问题是突现问题，具体而言就是连续时空结构如何从根本离散的理论中突现出来。该章提出，坚持“时空功能主义”立场可以化解一个据称会阻碍从离散基础重建连续时空的矛盾。正如这篇介绍性文章的前述章节所示，从因果集理论中突现出洛伦兹时空这一问题，无论是在运动学还是动力学层面，都是因果集合理论家群体关注的核心问题。这是一个双方共同关心的重要问题示例，有望促成有益交流与合作 [58]。该章的另一个研究焦点是时间哲学。这一点同样得到因果集合理论家和哲学家的共同关注。有一场争论围绕经典顺序增长模型中的生成过程是否以及在何种意义上构成了物理生成——也就是物理时间流逝——展开，因果集合理论家持上述观点。Wüthrich 在该章中从怀疑论立场出发提出，经典顺序增长模型中的生成类似于连续广义相对论背景下的“世界线生成”。这场争论的核心在于如何理解物理学与数学中“过程”概念的含义，它是双方富有成效的交流与思想碰撞的典型示例，这对该理论的未来发展只会起到积极作用。

Cross-References

交叉引用

Causal Set Cosmology

因果集合宇宙学

Computer Simulations of Causal Sets

因果集合的计算机模拟

Covariant Growth Dynamics

协变增长动力学

Entanglement Entropy and Causal Set Theory

纠缠熵与因果集合论

Estimating the Manifold Dimension of Causal Sets

因果集合流形维度估算

- Interacting Quantum Scalar Field Theory on a Causal Set

  - 因果集合上的相互作用量子标量场论

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论因果集合中的视界分子与熵

Quantum Field Theory on Causal Sets

因果集合上的量子场论

The Philosophy of Causal Set Theory

因果集合论的哲学

- Toward the Emergence of Continuum Spacetime in Causal Set Theory

  - 通向因果集合论中连续时空的涌现

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